

## **Historic, archived document**

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PROBLEM 1. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function such that  $f(x+y) = f(x) + f(y)$  for all  $x, y \in \mathbb{R}$ . Suppose also that  $f$  is continuous at  $x=0$ . Show that  $f$  is linear, i.e.  $f(x) = cx$  for some constant  $c \in \mathbb{R}$ .

SOLUTION. We first show that  $f$  is additive, i.e.  $f(x+y) = f(x) + f(y)$  for all  $x, y \in \mathbb{R}$ . This is given by the hypothesis.

Next, we show that  $f$  is homogeneous, i.e.  $f(ax) = af(x)$  for all  $a \in \mathbb{R}$  and  $x \in \mathbb{R}$ . For this, we first show that  $f(nx) = nf(x)$  for all  $n \in \mathbb{Z}$  and  $x \in \mathbb{R}$ . This is done by induction on  $n$ .

For  $n=1$ , we have  $f(1x) = f(x) = 1f(x)$ .

PROBLEM 2. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function such that  $f(x+y) = f(x)f(y)$  for all  $x, y \in \mathbb{R}$ . Suppose also that  $f$  is continuous at  $x=0$ . Show that  $f(x) = e^{cx}$  for some constant  $c \in \mathbb{R}$ .

SOLUTION. We first show that  $f$  is multiplicative, i.e.  $f(xy) = f(x)f(y)$  for all  $x, y \in \mathbb{R}$ . This is given by the hypothesis.

Next, we show that  $f$  is exponential, i.e.  $f(x) = e^{cx}$  for some constant  $c \in \mathbb{R}$ .

For this, we first show that  $f(1) = e^c$  for some constant  $c \in \mathbb{R}$ . This is done by evaluating  $f(1)$  at  $x=0$ .

PROBLEM 3. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function such that  $f(x+y) = f(x) + f(y)$  for all  $x, y \in \mathbb{R}$ . Suppose also that  $f$  is continuous at  $x=0$ . Show that  $f(x) = cx$  for some constant  $c \in \mathbb{R}$ .

SOLUTION. We first show that  $f$  is additive, i.e.  $f(x+y) = f(x) + f(y)$  for all  $x, y \in \mathbb{R}$ . This is given by the hypothesis.

Next, we show that  $f$  is homogeneous, i.e.  $f(ax) = af(x)$  for all  $a \in \mathbb{R}$  and  $x \in \mathbb{R}$ . This is done by induction on  $a$ .

For  $a=1$ , we have  $f(1x) = f(x) = 1f(x)$ .

PROBLEM 4. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function such that  $f(x+y) = f(x)f(y)$  for all  $x, y \in \mathbb{R}$ . Suppose also that  $f$  is continuous at  $x=0$ . Show that  $f(x) = e^{cx}$  for some constant  $c \in \mathbb{R}$ .

















I hereby certify that the above is a true and correct copy of the original as the same appears in the records of the County of [ ] State of [ ]

Witness my hand and seal of office this [ ] day of [ ] 19[ ]

Notary Public for the State of [ ]

*[Signature]*

